Original Research Article

Recurrently – Automatons Methods in the Analysis of the Laws of Functioning of Discrete Dynamical Systems

Anton Epifanov[1*]

[1]Institute of Precision Mechanics and Control Science of RAS, Rabochaya 24, Saratov, Russia

*Corresponding Author: Anton Epifanov
Institute of Precision Mechanics and Control Science of RAS, Rabochaya 24, Saratov, Russia

Abstract: The paper considers the laws of functioning of discrete dynamical systems. The special theoretical apparatus of geometrical images of automata is used for formal representation of the laws of functioning. Specific processes of system functioning and the law of functioning of the system as a whole are defined by geometric curves and numerical sequences. The spectrum of dynamic parameters of the recurrence definition of numerical sequences is used to build estimates of the complexity of processes and laws of functioning.

Keywords: Discrete deterministic dynamical system, automaton, geometrical image of automata mapping, sequence, complexity estimation.

INTRODUCTION

One of fundamentals, making mathematical models of discrete dinamical systems are the algorithms, realized by system according to its target mission. For an algorithmically solvable class of problems there is an infinite set of algorithms (solving a class of problems), which can be ordered on complexity. Realization of algorithms is generally connected with complexity of algorithm in the system, defining the major indicators: performance, a memory size, reliability, expenditure of energy etc. Number of variants of concept of complexity is sufficiently great and continues to increase in works of many researchers. For example, estimations of algorithms on their belong to NP and P classes (detail review see, for example, in work [1] and one of contemporary papers [2], in which Radoslaw Hofman show, that P not equal NP), Kolmogorov complexity [3], complexity from below, from above, complexity on the average, bit complexity (one of basic work in this area is [4]), multiplicate complexity, algebraic complexity, there are very large amount of works on asymptotical estimations of complexity (see, for example works [5, 6]), etc.

The fundamental concepts of complexity - the class of P-problems solvable in polynomial time by deterministic Turing machines, and the class of NP-problems solvable in polynomial time by non-deterministic Turing machines, are related to automatons (Turing machines). Along with the general "algorithmic" understanding of complexity in the theory of automata, particular variants of complexity indicators were investigated: the number of states of the automaton, the lengths of the input and output sequences, the number of connectivity components in the structure of the automaton, the complexity of the structure of the structural automaton, etc.

The problem of estimating the complexity of automata was investigated by many authors immediately after the introduction of automata models. John von Neumann devoted a number of sections of his work "The Theory of Self-Reproducing Automata" [7] to problems of estimating the complexity of automata: "The role of high and very high complexity (including the role of complexity and the need for appropriate theoretical justification, etc.); Re-evalution of the problems of complicated automata - problems of hierarchy and evolution." These facts further confirm the relevance and importance of research in the field of assessing the complexity of the laws of operation of discrete dynamical systems.

The estimation of the complexity of structural automata by the number of their constituent elements has become widespread. Such estimates change significantly with changes in the element base and synthesis method.

According to one of the basic hypotheses of algorithm theory (which is called the Church-Turing thesis in the special literature), if a problem has a solution (there is an algorithm for solving it), then there exists a Turing machine that solves that problem. According to this hypothesis the class of all algorithms is equal to the class of Turing machines. In its turn, a Turing machine is an extension of a finite deterministic automaton (finite state machine - FSM) whose class of transformations is bounded by the number of states, inputs and outputs. If we remove the restriction on finiteness of the number of states of the automaton (which was done by Tverdokhlebov V. A. by introducing geometrical images of laws of automata functioning [8]), the Turing machine can be represented as an automaton with an infinite number of states, which reads and writes information from the cells of the tape. A.N. Kolmogorov and V.A. Uspenskii [9] note: "... constructing a theory of algorithms on the model of the theory of computing machines requires in any case some idealization of the notion of 'machines'. ... All the idealization necessary for the transition from real computing machines to mathematical algorithms consists in the assumption of an unlimited volume of "memory" of the machine.(p.6)".

In connection with the development of areas of application of the theory of automata, it turned out that for real systems of large dimension, the assignment of automata models with tables, matrices, graphs, and logical equations is practically ineffective. One of the ways to expand the field of application of the theory of automata was the research of Academician of the Russian Academy of Natural Sciences V.A. Tverdokhlebov, in which, since 1993, is considered the representation of the laws of functioning of automata, given by discrete symbolic next state and output functions, by continuous numerical structures.

For this, the automata mapping is assumed to be a set of points with numerical coordinates and the laws of functioning are represented by broken lines, the vertices of which are located on analytically specified curves. This approach to defining of automata as well as some methods of analysis, synthesis and recognition of automata by their geometric images are used and developed.

The geometric approach used in this work to define the laws of the functioning of automata ideologically intersects with the research of other scientists [10-15], but has fundamental differences. For example, in [10] Benjamin Steinberg also considers a geometric approach to automata theory, but in the context of its use in combinatorial group theory, to solve various problems on the overlap between group theory and monoid theory. In [11] finite automatons are used in problems of encoding and compressing images, as well as in considering regular ω-languages (sets of infinite words defined by finite automata).

The work [12], which studies discrete-event systems, is of great interest, although it also uses symbolic forms of defining automata models of systems.

Quite interesting are the results of [13], which uses a geometric approach to describe such properties, within the framework of which the output function can be characterized by its polygon in the unit square $[0, 1] \times [0, 1]$, in [13] also investigated the properties of such polygons and their relationship with the properties of the used output function are investigated.

Among various approaches to estimating the complexity of processes, algorithms, laws of functioning of automata, and implementations of these laws, the approach in which the geometrical representation of automata behavior is used was chosen for the study. The justification for this choice is that the setting of the laws of automata functioning in a geometrical way is uniquely determined by the sequence of second coordinates of points in the geometrical image; in the representation of algorithms by Janov schemes, algorithms are also defined by linear structures, i.e. sequences; implementations of algorithms for specific input data correspond to a sequence of action operators and a sequence consisting of initial, intermediate, and final data. Recurrence description of sequences gives a complete and deep characteristic of the mutual disposition of elements in a sequence: it determines the functional dependence of an element in the sequence on the subsequence of elements that immediately precedes it. The numerical index at the recurrence-based definition of a sequence is the length of the unambiguously matched element of the subsequence that precedes it. On the basis of these properties of sequence definition by recurrence forms, V. A. Tverdokhlebov proposed 5 levels of spectrum that consist of numerical indices of the lowest order of recurrence form; numerical indices of lengths of initial sequence segments defined by recurrence forms of different orders; number of changes of recurrence forms that define sequence by sequences of recurrence forms of fixed order. The other levels of the spectrum characterize the lengths of subsequences defined by recurrence forms of different orders and characteristics of recurrence definitions taking into account weights for sets of arguments in the recurrence forms used.

The scientific novelty of this work consists precisely in the use of special theoretical apparatus of geometrical images of automata models of systems and the use of recurrence methods to analyze structures (numerical sequences) that uniquely define geometrical images. The analysis of open sources of literature allows us to conclude that similar studies conducted by the author have not been conducted before.

1. Mathematical Apparatus
Geometric images of the laws of the functioning of automata Recognition of geometric patterns of automatic models of systems

Geometric γ_s of the laws of operation (see [8]) (the next-state function $\delta : S{\times}X{\to}S$ and output functions $\lambda: S{\times}X{\to}Y$) of an initial finite state machine $A_s = (S, X, Y, \delta, \lambda)$ with sets of states' S, input signals X and output signals Y is determined based on the introduction of a linear order ω in the automata mapping $\rho'_s = \bigcup_{p \in X^*}\{(p, \lambda'(s, p))\}$,

where $\lambda'(s, p) = \lambda(\delta(s, p'), x)$, $p = p'x$. An automata mapping ρ'_s (set of pairs) is ordered by the linear order ω, defined based on the order ω_1 on X^* and given by the following rules:

Rule 1. A certain linear order ω_1 is introduced on the set X (which we will denote $\prec_1$);

Rule 2. The order ω_1 on X extends to a linear order on the set X^*, assuming that

For any words $p_1, p_2 \in X^*$ of unequal length ($|p_1| \neq |p_2|$) $|p_1| < |p_2| \to p_1 \prec_1 p_2$;

For any words $p_1, p_2 \in X^*$ for which $|p_1| = |p_2|$ and $p_1 \neq p_2$, their ratio in the order ω_1 repeats the ratio of the nearest non-coinciding letters of the words p_1 and p_2 to the left.

The order ω'_2 on the set of words Y^* is defined similarly.

After introducing the linear order ω_1 on the set X^* we obtain a linearly ordered set $\rho_s = (\rho'_s, \omega'_1)$, where ω'_1 is the order on ρ'_s induced by the order ω_1 on X^*.

Defining the linear order ω_2 on the set Y and placing the set of points in the coordinate system with the abscissa axis (X^*, ω_1) and the ordinate axis (Y, ω_2), we obtain a geometric image of the laws of functioning of an initial finite state machine $A_s = (S, X, Y, \delta, \lambda, s)$. It should be noted that the linear orders ω_1 on X^* and ω_2 on Y are generally independent. This means that the specific form of the a geometric image of the laws of functioning of an initial finite state machine $A_s = (S, X, Y, \delta, \lambda, s)$ depends on the chosen orders ω_1 and ω_2. Other variants of linear orderings on X^* are also possible (see, for example, [8]). In this paper, the study of the laws functioning of an initial finite state machine is carried out using the order ω_1 on X^* defined above. Linear orders ω_1 and ω_2 allow to replacing elements of the sets X^* and Y by their numbers $r_1(p)$ and $r_2(p)$ in these orders. As a result, two forms of geometric images are determined, firstly, as a symbolic structure in a coordinate system D_I, and secondly, as a numerical structure in a coordinate system with integer or real positive semiaxes.

From the geometric image γ_s of the automaton A_s is extracted sequence of second coordinates of points of the geometric image, which one-to-one corresponds to the complete geometric image (for a fixed order on the set X^* and the value m = |X|). As a result, the laws of the automaton functioning (that is, the phase picture) and the specific processes of the automaton functioning (that is, the phase trajectories) turn out to be one-to-one determined by the sequence of the second coordinates of the points of the geometric image. This allows us to consider an arbitrary sequence of elements from a finite set as a sequence of the second coordinates of points of a geometric image and, therefore, as setting the laws of the automaton functioning.

The representation of a geometric image γ_s as a numerical structure allows to use the apparatus of continuous mathematics in the formulations and methods of solving problems: setting the laws of the functioning of automata by numerical equations, using numerical procedures, interpolation and approximation of partially given laws of functioning, etc. The geometric image γ_s completely determines the laws of the automaton functioning, that is, the entire phase picture of the connections of the input sequences with the output signals. Specific variants of the functioning processes, that is, phase trajectories, have geometric images $\gamma_s(p), p \in X^*$, in the form of γ_s sections along individual points. Geometric images can also be defined by numerical, rather than symbolic, equations.

2. RESEARCH METHODS

Spectrum of recurrent definition dynamic parameters as a means of sequence structure complexity estimation.

In [8] the concept of a spectrum was introduced and developed, which characterizes the structure of a sequence, which can be considered as a way of definition of a geometric image of the laws of functioning.

Let be $U = \{u_1, u_2, ..., u_k\}$ a finite set and ξ a sequence of elements from the set U: $\xi = \langle u(1), u(2), ..., u(t), ... \rangle$. The sets of all finite sequences, all finite sequences of length v and infinite sequences of elements from the set U will be denoted by U^*, U^v, U^∞, respectively. The spectrum $\Omega(\xi)$ of dynamic characteristics of the sequence $\xi \in U^* \cup U^\infty$ has a hierarchical structure consisting of the levels $\Omega(\xi) = (\Omega_0(\xi), \Omega_1(\xi), \Omega_2(\xi), \Omega_3(\xi), \Omega_4(\xi))$. Each specific implementation option (represented by parameter values) of any level $\Omega_i(\xi)$ defines a partition of each of the sets U^*, U^v, U^∞ into subsets according to the coincidence properties of the characteristics corresponding to the level. We will consider subsets of such a partition as equivalence classes of sequences.

Definition 1. For any sequence $\overline{\overline{\xi}} \in U^v$, the least order of the recurrent form that defines the sequence $\overline{\overline{\xi}}$ will be denoted $m_0(\overline{\overline{\xi}})$.

Definition 2. For any sequence $\overline{\overline{\xi}} \in U^v$ and $m \in N^+$, where $1 \leq m \leq m_0(\overline{\overline{\xi}})$, the longest length of the initial segment of the sequence $\overline{\overline{\xi}}$ determined by the recurrence form of order m will be denoted $d^m(\overline{\overline{\xi}})$.

Definition 3. For any sequence $\overline{\overline{\xi}} \in U^v$ and $m \in N^+$, where, $1 \leq m \leq |\overline{\overline{\xi}}| - 1$, the number of shifts of recurrent forms of order m required in determining the sequence $\overline{\overline{\xi}}$ will be denoted $r^m(\overline{\overline{\xi}})$.

Definition 4. For any sequence $\overline{\overline{\xi}} \in U^v$ and $m \in N^+$, where $1 \leq m \leq m_0(\overline{\overline{\xi}})$ and j, where $1 \leq j \leq r^m(\overline{\overline{\xi}})$, the length of the j-th segment in the definition of a sequence $\overline{\overline{\xi}}$, will be denoted $d_j^m(\overline{\overline{\xi}})$.

Using the introduced notation, we define the spectrum of parameters characterizing the sequence as the following structure:

$$\Omega_0(\overline{\overline{\xi}}) = \langle m_0(\overline{\overline{\xi}}) \rangle;$$
$$\Omega_1(\overline{\overline{\xi}}) = \langle d^1(\overline{\overline{\xi}}), d^2(\overline{\overline{\xi}}), ..., d^\alpha(\overline{\overline{\xi}}) \rangle;$$
$$\Omega_2(\overline{\overline{\xi}}) = \langle r^1(\overline{\overline{\xi}}), r^2(\overline{\overline{\xi}}), ..., r^\alpha(\overline{\overline{\xi}}) \rangle;$$
$$\Omega_3(\overline{\overline{\xi}}) = \langle \Omega_3^1(\overline{\overline{\xi}}), \Omega_3^2(\overline{\overline{\xi}}), ..., \Omega_3^\alpha(\overline{\overline{\xi}}) \rangle$$

Where $\alpha = m_0(\overline{\overline{\xi}})$ and $\Omega_3^j(\overline{\overline{\xi}}) = \langle d_1^j(\overline{\overline{\xi}}), d_2^j(\overline{\overline{\xi}}), ..., d_{n_j}^j(\overline{\overline{\xi}}) \rangle$ (n_j – the number of the last segment in the definition of a sequence $\overline{\overline{\xi}}$ as a sequence of segments defined by individual recurrent forms of order j);

$\Omega_4(\overline{\overline{\xi}}) = \Theta(\Omega_3(\overline{\overline{\xi}}))$, where Θ is the operator of replacing in $\Omega_3(\overline{\overline{\xi}})$ the lengths of the segments by the weights of the used recurrent forms for determining the segments.

The fourth level $\Omega_4(\overline{\overline{\xi}})$ of the spectrum $\Omega(\overline{\overline{\xi}})$ adds to the characteristics in the previous levels an assessment of the complexity of the rules and the options for using the rules.

The construction of the spectrum on the basis of the hierarchical structure was made with the aim of successively deepening and expanding the characteristics of the properties of the sequence when moving from the previous level to the next. Such structure allows one to reduce the set of parameters used, if the goal of analysis is achieved, and to be limited to the corresponding initial fragments of the spectrum.

Spectrum levels are hierarchically constructed by adding new parameters to the spectrum. In this regard, the equivalence of sequences is represented through t - equivalences determined by the level $\Omega_t, 0 \leq t \leq 3$.

3. RESULTS
Construction of complexity estimates of automatons, defined by sequences from OEIS

In this section of the paper, the properties of fundamental mathematical quantities, defined approximately in the form of sequences are investigated on the basis of the spectrum of dynamic parameters. In view of the fact that a geometric image is mutually unambiguously determined by a sequence of second coordinates of its points, the section investigates the properties of the laws of functioning on the basis of an analysis of the properties of numerical sequences. We consider the set of sequences of length from 80 to 1000 signs represented in the bank [16] that determine approximations of fundamental mathematical quantities. To classify and assess the complexity of the sequences, a spectrum of dynamic parameters characterizing the complexity of the rules of sequence generation is used. A spectrum is constructed for each sequence and the set is divided into classes of equivalent sequences based on the coincidence of the indicators of the constructed spectrums.

The basic model of discrete deterministic dynamical systems is finite deterministic automata. A comparison of the complexity of such automata can be made on the basis of a comparison of mathematical structures representing the specificity of the laws of automata functioning. Geometrical images of the laws of automata functioning can be used as such structures. The geometric image is constructed on the basis of, first, a linear ordering of elements of the automata mapping, and, second, the extraction of a sequence of second coordinates of points from a linearly ordered automata mapping. A comparison in terms of the complexity of the laws of functioning of automata is made on the basis of a comparison of the spectrums of the sequences. Based on the fact that no restrictions are imposed on the structure of sequences and that it can be any sequence of elements from a finite set, the method of comparing models in terms of complexity admits interpretation on any, in particular numerical, sequences. From a substantive point of view, spectra determine the complexity of sequences taking into account the properties of different variants of recurrence definitions of sequences. In variants of such definitions, numerical indicators are orders of recurrence forms defining individual segments of sequences and number of changes of recurrence forms used to define segments of a sequence. The levels of the spectrum are built hierarchically by adding new parameters to the spectrum. In this regard, the equivalence of the sequences is represented through t - equivalences defined by the level Ω_t, $0 \leq t \leq 3$.

In this section the bank of fundamental mathematical quantities, which was originally formed by N. Sloan and currently consists of more than 300000 integer sequences, is investigated using the spectrum of dynamic parameters. This bank is called "The On-Line Encyclopedia of Integer Sequences" (the abbreviation OEIS is used in this paper).

The results of the OEIS bank studies are:
1. Splitting the set of sequences under consideration into classes based on spectrum indices, i.e. into classes of sequences that have the same complexity of mutual arrangement of elements in the sequence;
2. Constructing for each sequence of characteristics used to estimate the complexity of the sequence and dividing the set of sequences by the coincidence of the values of characteristics.

In this paper, the symbol Ξ is used to denote the initial set of sequences. The conducted research of the sequence bank includes the following steps:

1. From the initial set of sequences Ξ is extracted a subset $\tilde{\Xi}$, consisting only of those sequences whose elements are positive integers from 0 to 9, which does not violate the illustration of the generality of the method for comparing sequences by the performance of Ω;
2. Constructing the set Ψ, whose elements are the initial segments of length 80 of sequences from the set $\tilde{\Xi}$, whose length l , $l \geq 80$;
3. Calculating for each element of the set Ψ the numerical values of the dynamic parameters at the levels Ω_0 - Ω_3 of the spectrum Ω;
4. Construction of partitions P_0, P_1, P_2, P_3 of the set of sequences Ψ into subclasses of equivalent sequences by the coincidence of the spectrum values and analysis of obtained subclasses.

After analyzing the sequences presented in the bank [16], a set Ψ consisting of 8594 sequences of length 80, whose elements are only integers from 0 to 9, is constructed. Partitions P_0, P_1, P_2, P_3 of the set Ψ into classes of sequences equivalent in terms of spectrum indicators are constructed.

Obviously, when considering the set of sequences Ψ , each of which has a length of 80 characters, in the division of P_0 of this set into classes of equivalent sequences by coincidence of the values of the level Ω_0, the number of classes must lie in the interval from 1 to 79 inclusive, because $1 \leq m_0 \leq 79$.

Consequently, in the partition P_0 of the set Ψ the maximum value of the number of classes of equivalent sequences can be equal to 79. In the analysis of the set of sequences Ψ it is observed that there exist values of m_0,

$1 \le m_0 \le 79$, which are not the minimum order of the recurrence form necessary to determine any of the sequences of the set Ψ.

In fact, the number of classes in the partition P_0 of the set Ψ is 60. Thus there are exactly 19 different values of m_0, which are not the minimum order of the recurrence form needed to determine any of the sequences of the set Ψ. Table 1 shows the characteristics of the partitions of the set of sequences Ψ. A significant increase in the number of classes of equivalent sequences (by 2 orders of magnitude) is observed in the construction of partitioning P_1. At the same time, the value of the power of the maximum class decreases.

Table 2 lists some classes in the partitions P_0 and P_3 of the set Ψ (in abbreviated form: only the sequence number in OEIS and its description is given).

Table 1: Information about partitions P_0, P_1, P_2, P_3 of set Ψ of sequences by indicators of sectrum Ω on levels $\Omega_0 - \Omega_3$

Characteristics	P_0	P_1	P_2	P_3
Number of classes in the division	60	6262	7806	7972
Maximum class capacity	2100	346	49	49
Minimum class capacity	1	1	1	1

For each sequence of the classes of equivalent sequences given in the table are indicated:
1. Its number in open database of integer sequences The On-Line Encyclopedia of Integer Sequences (hereinafter abbreviated as OEIS);
2. Description and possible interpretation of the sequence in the original, also extracted from [16];
3. The level of the spectrum by which the sequences are equivalent.

Table 2: Classes of equivalent sequences in partitions P_0 and P_3 of the set Ψ *(abbreviated information)*

Number of sequence in OEIS	Description of a sequence in OEIS	Spectrum level, by indicator of which sequences are equal
A019765	Decimal expansion of 2*E/7	Ω_0
A098592	Number of primes between n*30 and (n+1)*30.	
A001113	Decimal expansion of e.	
A023976	First bit in fractional part of binary expansion of 9-th root of n.	Ω_0
A072609	AC-range corresponds to 0, while DC-range labeled by 1.	
A095907	Digits in the concatenation of strings formed from a previous string by substituting "01" for "0" and "011" for "1" simultaneously at each occurrence. Start with [0].	
A071029	Triangle read by rows giving successive states of cellular automaton generated by "rule 22".	
A071040	Triangle read by rows giving successive states of cellular automaton generated by "rule 214".	
A057427	Sign(n): a(n) = 1 if n>0, = -1 if n<0, = 0 if n = 0.	Ω_3
A000007	The characteristic function of 0: a(n) = 0^n.	
A000038	Twice A000007	
A054977	a(0)=2, a(n)=1, n >= 1.	
A036453	a(n)=d[d[d[d[d[n]]]]], the 5th iterate of number-of-divisors function with initial value of n.	
A010873	The rightmost digit in the base-4 representation of n. Also, the equivalent value of the two rightmost digits in the base-2 representation of n, a(n) = n mod 4	Ω_3
A010883	a(n) = 1 + (n mod 4)	
A070402	2^n mod 5.	
A091086	a(n)=2-(-1)^n-cos(pi*n/2)-2sin(pi*n/2)	
A077750	a(4n-3)=0, a(4n-2)=2, a(4n-1)=6, a(4n)=4, n>0	
A055874	a(n) = largest m such that 1, 2, ..., m divide n.	Ω_3
A007978	Least non-divisor of n.	

The geometrical approach used allows us to investigate the properties of the laws of functioning of discrete deterministic dynamical systems on the basis of the analysis of the properties of sequences of elements of a finite set. The

constructed spectrums characterize the complexity of the sequences under study from the point of view of their generation by recurrence forms of different orders. The spectrum characterizes all variants of bases of recurrence forms, which allows us to use the spectrums as characteristics of laws.

To present a class of equivalent sequences with full information about each member of the class, even if the power of the class is 10, requires several pages of text. In view of this limitation, only a few classes were chosen by the author for inclusion in this paper.

The choice of classes for illustration was made on the basis of "popularity" and "prevalence" of its elements.

Table 3 below shows some class in the partitions P_3 of the sequence set Ψ, including an approximation of the number π (Table 3).

Table 3: The class of equivalent sequences by indicators of of the third level Ω_3 of the spectrum Ω, containing an approximation of the number π

Number of sequence in OEIS	Description of a sequence in OEIS
A000796	Decimal expansion of Pi. 3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3,2,3,8,4,6,2,6,4,3,3,8,3,2,7,9,5,0,2,8,8,4,1,9,7,1,6,9,3,9,9,3,7,5,1,0,5,8,2,0,9,7,4,9,4,4,5,9,2,3,0,7,8,1,6,4,0,6,2,8,6,2,0,8,9
A059833	"Madonna's Sequence": add 1 (mod 10) to each digit of Pi. 4,2,5,2,6,0,3,7,6,4,6,9,0,8,0,4,3,4,9,5,7,3,7,5,4,4,9,4,3,8,0,6,1,3,9,9,5,2,0,8,2,7,0,4,0,0,4,8,6,2,1,6,9,3,1,0,8,5,0,5,5,6,0,3,4,1,8,9,2,7,5,1,7,3,9,7,3,1,9,0
A089250	Add 2 (mod 10) to each decimal digit of Pi. 5,3,6,3,7,1,4,8,7,5,7,0,1,9,1,5,4,5,0,6,8,4,8,6,5,5,0,5,4,9,1,7,2,4,0,0,6,3,1,9,3,8,1,5,1,1,5,9,7,3,2,7,0,4,2,1,9,6,1,6,6,7,1,4,5,2,9,0,3,8,6,2,8,4,0,8,4,2,0,1
A030644	Decimal expansion of 10-Pi. 6,8,5,8,4,0,7,3,4,6,4,1,0,2,0,6,7,6,1,5,3,7,3,5,6,6,1,6,7,2,0,4,9,7,1,1,5,8,0,2,8,3,0,6,0,0,6,2,4,8,9,4,1,7,9,0,2,5,0,5,5,4,0,7,6,9,2,1,8,3,5,9,3,7,1,3,7,9,1,0

A study of sequences from the OEIS bank that included building spectrums for the extracted subset of sequences Ψ and classifying the set of fundamental mathematical sequences Ψ based on the matching of quantitative values of the spectrum indicators showed that the spectrum of dynamic parameters of recurrenceally determined numerical sequences can be used to classify arbitrary sequences of elements from a finite set and to classify laws of functioning of discrete determined dynamical systems.

4. DISCUSSIONS AND POSSIBLE DIRECTIONS FOR FURTHER RESEARCH

The results presented in the article show the possibility of practical use of the spectrum of dynamic parameters to determine the properties of the laws of functioning of discrete deterministic dynamic systems on the basis of the study of the properties of numerical sequences that mutually unambiguously determine the laws of functioning.

It is of independent interest to study the properties of binary characteristic sequences (interpreted as sequences of second coordinates of points of geometrical images of automata models of systems) that carve numbers with specific properties in a natural number series using the spectrum of dynamic parameters described above.

One of the important and open questions in both number theory and cryptography is the question of the structure of the distribution of prime numbers in natural numbers. For finding prime numbers from more than 100,000,000 and 1,000,000,000 decimal digits EFF (Electronic Frontier Foundation - a nonprofit human rights organization founded in July 1990 in the United States to protect the rights embodied in the Constitution and the Declaration of Independence in connection with the advent of new communication technologies) awarded cash prizes of $150,000 and $250,000, respectively. Earlier, the EFF has already awarded prizes for finding prime numbers from 1,000,000 and 10,000,000 decimal digits. In this regard, one of the possible directions for the further research can be the construction and analysis of spectrums for characteristic sequences of large length, which distinguish prime numbers in a natural numbers, as well as characteristic sequences reflecting combinations of the properties of prime numbers, Mersenne numbers, Lucas numbers, Woodall (or Riesel) numbers, numbers Cullen, Proth numbers, Mills numbers, etc. Perhaps, in this direction it will be possible to identify new properties of prime numbers that are interest.

5. CONCLUSIONS

The results outlined in the paper show the possibility of practical use of the spectrum of dynamic parameters to determine the properties of the laws of functioning of discrete deterministic dynamical systems by investigating the properties of numerical sequences that mutually unambiguously determine the laws of functioning. The basic model of discrete deterministic dynamical systems is finite deterministic automata. A comparison in terms of the complexity of such automata can be made on the basis of a comparison of mathematical structures representing the specificity of the laws of functioning of the automaton. Geometric images of the laws of automata functioning can be used as such structures. The geometric image is built on the basis, first, of a linear ordering of elements of the automata mapping, and, second, of the extraction of a sequence of second coordinates of points from the linearly ordered automata mapping. A comparison in terms of the complexity of the automata functioning laws is based on a comparison of the spectra of the sequences. Based on the fact that no restrictions are imposed on the structure of sequences and it can be any sequence of elements from a finite set, the method of model complexity comparison admits interpretation on any, in particular numerical, sequences. From the substantive point of view, spectra define the complexity of sequences taking into account the properties of different variants of recurrence definitions of sequences. In the variants of such definitions, the numerical indicators are the orders of recurrence forms defining individual segments of sequences and the number of changes of recurrence forms used to define the segments of the sequence. Application of a special spectrum of dynamic parameters in the analysis of structures of mutual arrangement of elements in sequences has allowed us to obtain new results in the analysis of discrete dynamic systems and, moreover, has allowed us, by the example of sequences defining fundamental mathematical quantities, to carry out complexity estimates and classify by complexity the laws of functioning of systems.

ACKNOWLEDGEMENT

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